Preface

Partial differential equations play an important role in mathematical physics. Usually some initial conditions or boundary conditions are imposed on the equations, and we are dealing with initial value problems, boundary value problems, or initial-boundary value problems. There are some different points of view to classify the problems: Linear or nonlinear, first order equations or higher order equations, steady problems or evolution problems, interior problems or exterior problems. The first three are according to the properties of the equations, and the last one is according to the properties of the underlying physical domains. For example, the flow problem in a chamber is an interior problem, and the flow problem around an aircraft is an exterior problem.

All partial differential equations possess infinite number of degrees of freedom. To solve a problem numerically, one should approximate the problem so that only a finite number of degrees of freedom are solved in real computation. As a result a truncation error appears in every numerical scheme. This fact is the same for both interior and exterior problems. However when one intends to solve an exterior problem, one faces another kind of difficulty that the domain is infinitely large. Most approaches for the interior problems can not be applied to the exterior problems directly. The importance of exterior problems is obvious, and the challenge is serious.

There are two kinds of approaches to deal with the exterior problems: to truncate the domain, or to solve the problem directly on the infinite domain. The former ones are: the introducing of artificial boundary conditions, and the introducing of perfectly matched layers. The later ones are: the infinite element method, and the spectral method. While in the boundary element method both approaches are applied, depending on the degree of complexity of the domains.

The aim of this monograph is to provide a comprehensive study of different approaches. At the beginning we investigate the mathematical theory of the exterior problems of some typical partial differential equations, which provides the foundation of all numerical methods. Restricted by the space we only state the basic results. The readers can refer to the literature for the details. The main body of the book is on some traditional and new methods. Each of them is effective, and certainly restricted to some particular classes of problems. One omission of the book is that no numerical example is included, because there are too many in the literature, and it is too difficult to choose a few, representing most of the problems and most of the methods. It is also due to the restriction of space. However I am not sure if it is appropriate to do so.

This is an area growing rapidly. Limited by the author's knowledge, some important approaches must have been ignored. There is a proverb in China:" Cast away a brick and attract a jadestone". I will be pleased to listen to all comments.

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