

Contents

<i>Preface</i>	v
1. Exterior Problems of Partial Differential Equations	1
1.1 Harmonic equation–potential theory	2
1.2 Poisson equations	12
1.3 Poisson equations–variational formulation	13
1.4 Helmholtz equations	17
1.5 Linear elasticity	25
1.6 Bi-harmonic equations	29
1.7 Steady Navier-Stokes equations -linearized problems . . .	35
1.7.1 Navier-Stokes equations	35
1.7.2 Stokes equations	36
1.7.3 Behavior of solutions at the infinity	39
1.7.4 Stokes paradox	41
1.7.5 Oseen flow	41
1.8 Steady Navier-Stokes equations	44
1.9 Heat equation	49
1.10 Wave equation	53
1.11 Maxwell equations	56
1.12 Darwin model	61
2. Boundary Element Method	71
2.1 Some typical domains	71
2.1.1 Harmonic equation	71
2.1.2 Bi-harmonic equation	75
2.1.3 Stokes equation	77

2.1.4	Plane elasticity	80
2.1.5	Helmholtz equation	82
2.2	General domains	85
2.3	Subdivision of the domain	93
2.4	Dirichlet to Neumann operator	96
2.5	Finite part of divergent integrals	98
2.6	Numerical approximation	103
2.7	Error estimates	108
2.8	Domain decomposition	113
2.9	Boundary perturbation	114
3.	Infinite Element Method	117
3.1	Harmonic equation-two dimensional problems	117
3.1.1	Infinite element formulation	117
3.1.2	Transfer matrix	120
3.1.3	Further discussion for the transfer matrix	127
3.1.4	Combined stiffness matrix	131
3.2	General elements	133
3.3	Harmonic equation-three dimensional problems	134
3.4	Inhomogeneous equations	136
3.5	Plane elasticity	138
3.6	Bi-harmonic equations	140
3.7	Stokes equation	142
3.8	Darwin model	147
3.9	Elliptic equations with variable coefficients	152
3.9.1	A homogeneous equation	152
3.9.2	An inhomogeneous equation	155
3.9.3	General multiply connected domains	158
3.9.4	Transfer matrices	161
3.10	Convergence	162
4.	Artificial Boundary Conditions	167
4.1	Absorbing boundary conditions	167
4.2	Some approximations	172
4.3	Bayliss-Turkel radiation boundary conditions	175
4.4	A lower order absorbing boundary condition	176
4.5	Liao extrapolation in space and time	178
4.6	Maxwell equations	178

4.7	Finite difference schemes	182
4.8	Stationary Navier-Stokes equations	183
4.8.1	Homogeneous boundary condition at the infinity .	183
4.8.2	Inhomogeneous boundary conditions at the infinity	186
4.8.3	A linear boundary condition	187
5.	Perfectly Matched Layer Method	191
5.1	Wave equations	191
5.2	Bérenger's perfectly matched layers	197
5.3	Stability analysis	201
5.4	Uniaxial perfectly matched layers	208
5.5	Maxwell equations	210
5.6	Helmholtz equations	212
6.	Spectral Method	217
6.1	Introduction	217
6.2	Orthogonal systems of polynomials	225
6.3	Laguerre spectral methods	230
6.3.1	Mixed Laguerre-Fourier spectral method	230
6.3.2	Spherical harmonic-generalized Laguerre spectral method	235
6.3.3	Generalized Laguerre pseudo-spectral method . .	237
6.3.4	Nonlinear equations	239
6.4	Jacobi spectral methods	241
6.5	Rational and irrational spectral methods	243
6.6	Error estimates	245
	<i>Bibliography</i>	251
	<i>Index</i>	265