Preface

This book is based upon the works of the author at Peking University during the last half-century. It concerns the topics of the qualitative theory of ordinary differential equations. The selection of material depends mainly upon the interests of the author. No rigorous attempt has been made to give the historic origin of the theory. The author would like to describe the following contents as the main feature of the book.

1) Chapter 1 begins with the existence and uniqueness theorem on the solution of the Cauchy problem. It provides fundamental exercises that are useful even in the advanced theory of differential equations. For example, we prove a general convergence theorem on difference methods for ordinary differential equations; that is, the numerical calculation of general difference method is convergent if and only if the solution of the corresponding Cauchy problem is unique. It means that the numerical calculation is divergent if Peano phenomenon happens to the Cauchy problem. On the other hand, it is proved that Peano's phenomena are dense among differential equations.

2) The global behavior of the solution to the Cauchy problem on infinite interval is considered in Chapter 2. According to the view-point of Poincaré, the solution on infinite interval may be continuous or discontinuous about the initial conditions, so it may be predictable or unpredictable. This leads to the Liapunov stability theory of motion.

The geometrical consideration of solutions yields the qualitative theory of differential equations, especially, with fruitful results for the autonomous differential equations (Chapter 3). On the other hand, it can be considered that the theory of dynamical systems is an abstract setting of the qualitative theory of ordinary differential equations (Chapter 5).

Finally, the theory of chaos is a natural consideration for the Liapunov unstable motions (Chapter 8). Following the current literature, we define the chaotic motion by means of the sensitive dependence on the initial conditions. Accordingly, we find a simple sufficient condition for chaotic motions, which implies that the C^1-flow on closed surface is chaotic if it has a dense orbit and finitely many (at least one) equilibrium points on the surface. The definition of chaotic motions implies obviously that all the motions in the chaotic set considered are Liapunov uniformly unstable. Therefore, it is possible that more complicated oscillations than "chaotic motions" may appear if Liapunov unstable motions coexist with Liapunov stable motions in some transitive invariant set.

3) Chapters 6 and 7 are concerned with the fixed point theorems, which are known as the main topological tools in nonlinear analysis. For example, the generalized Poincaré-Birkhoff twist theorem plays an important role for conservative systems. However, its area-preserving assumption is a severe restriction in application. We are interested in a flexible condition instead and thus obtain the bend-twist theorem for analytic maps (Chapter 7).

Using the bend-twist theorem, we prove that the dissipative super-linear Duffing equation

$$\ddot{x} + \varepsilon c\dot{x} + (ax + bx^3) = \varepsilon E \sin \omega t, \qquad (D)$$

has subharmonic motions of high order, where the generalized Poincaré-Birkhoff twist theorem does not work. In the history, the earliest strange attractor was discovered in the numerical analysis of (D) performed by the experiment in Tokyo University during the 1950's. According to Hayashi's viewpoint, the complicated strange attractor is caused by the "subharmonic motions", but his student Ueda considered the cause of "chaotic motions". Either way, the existence of subharmonic motions for the above dissipative equation (D) is now proved in this book.

4) Based on the collaboration work with F. Zanolin, we analyze the periodic Duffing equation of second order in Chapter 10. The attentive readers will find that a series of results was proved on this subject. On the other hand, Chapter 11 is devoted to the analysis of some special problems, which were solved by the author, as applications for the qualitative theory of differential equations.

The author is indeed indebted to his teaching work carried out over his years at Peking University, which has encouraged him to continue in his writing. At the same time, the preparation of this book was also facilitated by assistance from colleagues, in particular, Prof. Liu Bin and Prof. Wang Zaihong, and also Ed. Zhang Ji from the board of WSPC.