Preface

Since the second half of the twentieth century, the Riemannian and semi-Riemannian geometries have been active areas of research in differential geometry and its applications to a variety of subjects in mathematics and physics. Recent survey in Marcel Berger's book [15] includes the major developments of Riemannian geometry since 1950, citing the works of differential geometers of that time.

During mid 70's, the interest shifted towards Lorentzian geometry, the mathematical theory used in general relativity. Since then there has been an amazing leap in the depth of the connection between modern differential geometry and mathematical relativity, both from the local and the global point of view. Most of the work on the global Lorentzian geometry has been described in a standard book by Beem and Ehrlich [10] and in their second addition in 1996, with Easley. They concentrated on geodesic and metric completeness, the Lorentzian distance function, and the Morse index theory for Lorentzian manifolds. Ehrlich and his collaborators [33] are still actively working on the volume comparison theorems for Lorentzian manifolds, using warped product technique which was introduced by Bishop and O'Neill [17] in 1969.

In 1996, Duggal-Bejancu published a book [28] on the lightlike (degenerate) geometry of submanifolds needed to fill an important missing part in the general theory of submanifolds. That book included a series of papers on a specific technique of introducing a non-degenerate screen distribution to define the induced geometric objects such as linear connection, second fundamental form, needed to obtain the Gauss-Codazzi type equations for a lightlike submanifold. Since then several researchers have done further work on lightlike geometry by direct use of Duggal-Bejancu's technique and also, in general, there has been an increase in papers on the geometry and physics of null curves and hypersurfaces using several ways corresponding to a given problem.

The objective of this book is to present a comprehensive up to date information on the differential geometry of null curves and lightlike hypersurfaces.

The motivation comes from considerable new information on the geometry of these two interrelated topics and their use in mathematical physics. Indeed, see Ferrández *et al.* [35, 36, 37, 38, 39, 40, 41] on null curves, soliton solutions and relativistic particles involving the curvature of $3D$ null curves; Gutiérrez *et al.* [49, 50, 51, 52] on null conjugate points along null geodesics and Perlick [90, 91, 92, 93, 94] on a variational principle for light rays and many referred therein.

The works of all above cited researchers, along with the works of present authors (see Duggal [23, 24, 25, 26, 27] on globally null manifolds and null geodesics; Duggal-Jin [30] and Jin [67, 68, 69, 70, 71] on geometry of null curves), is the main source of inspiration in writing this book.

Moreover, these topics are suitable for those graduate students who know the theory of non-null curves and surfaces and are interested in their null counterparts. To the best of our knowledge, there does not exist any other text book covering the material included in this book which is just within the understanding (neither too high nor too elementary) of a graduate student. A senior level undergraduate course in differential geometry is the sole prerequisite. A fresh and improved version of the material appeared in [28, Chapters 3 and 4] is included to make this volume a self contained book. Our approach, in this book, has the following special features:

- The preliminaries are introduced as needed at the appropriate places. We expect that this approach will help the readers to understand each chapter independently without knowing all the prerequisites in the beginning.
- This is the first-ever graduate text book on null curves and hypersurfaces.
- The book contains a large variety of solved examples and exercises which range from elementary to higher levels.
- The sequence of chapters is arranged so that the understanding of a chapter stimulates interest in reading the next one and so on.

There are eight chapters whose subjects are clear from the contents. Equations are numbered within each chapter and its section. To illustrate this, we have introduced a triplet (a, b, c) for each equation such that a, b and c stand for the chapter, the section and the number of equation in that section accordingly. Likewise, say in chapter a, theorem $b.c$ means theorem number c in section b. Each chapter is accompanied by a set of notes of background material (may not be familiar to some readers) followed by a set of typical exercises.

Overall this book is aimed at graduate students, research scholars and faculty interested in differential geometry. As a text book, it is suitable in sequence for the following two consecutive graduate semester courses meeting 3 hours a week:

First semester. Chapters 1, 2, 3 and 4. Prerequisite. Any senior undergraduate course in Differential Geometry.

Second semester. Chapters 5, 6, 7 and 8 and reading of related research papers.

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