Preface

This book is divided into two parts. The first part is about Sign-Changing Yamabe-type problems. A Morse Lemma at infinity, under reasonable basic conjectures, is proved for such problems. This work is an attempt to define a new area of research for nonlinear analysts. We have tried in it to provide a family of estimates and techniques with the help of which the problem of finding infinitely many solutions to these equations on domains of $\mathbb{R}^3$ can be studied.

Our estimates and our work is a "cas d'ecole" in that we work on $\mathbb{R}^3$ or S^3, a framework where solutions are known to exist, in fact in infinite number; and we have chosen to study the asymptots generated by these solutions and their combinations under the action of the Conformal Group. This work **could** also be useful for other variational problems such as Einstein or Yang-Mills equations.

The second part of this book is about Contact Form Geometry via Legendrian curves. Given a three-dimensional compact orientable manifold M^3 and a contact form α on it, we have assumed in earlier works the existence of a "dual" contact form $\beta, \beta = d\alpha(v, \cdot)$, with the same orientation than α and we have introduced the variational problem $\int_0^1 \alpha_x(\dot{x})dt$ on $C_\beta = \{x \in H^1(S^1, M) | \beta_x(\dot{x}) \equiv 0\}$. We have defined a homology related to the periodic orbits of the Reeb vector-field ξ of α on C_β.

We prove in this framework two main results. First, we establish that the hypothesis that β is a contact form with the same orientation than α is not essential. The techniques involved in order to prove such a result (on a typical example) have the definite advantage that they are quantitative: as we allow regions where β is no longer a contact form with the same orientation than α, we track down the modifications of the variational problem and we provide bounds on a key quantity (denoted τ) as we introduce a large amount of rotation for $\ker \alpha$ along the orbits of v near these areas.

We then move to prove a compactness result about the flow-lines of this variational problem which originate at a periodic orbit of ξ. This — still slightly imperfect — compactness result indicates that all flow-lines originating at periodic orbits go to periodic orbits (at least if the difference of Morse indexes is 1), unless the number of zeros of the v-component of $\dot{x}$, the tangent vector to the curves under deformation, drops.

No critical point at infinity (asymptot)interferes with this homology. We strongly suspect that this homology is, in the case of the standard contact structure of S^3, the homology of $P\mathbb{C}^\infty$. We expect that we will be able to compute this homology in some easy cases at least.

We had been searching for a long time for such a result. This work entitled "Compactness" will be published independently by the first author and dedicated to his long time friend and collaborator Haim Brezis for his sixtieth birthday. Both directions of research i.e. Conformal Geometry, Einstein equations, Yang-Mills equations on one hand, Contact Form Geometry on the other hand, are also studied by other techniques due to "hard-chore" Geometry and Symplectic Geometry.

In fact, Geometers have always been our "co-area researchers". We view these areas which we have contributed to define — for Contact Form Geometry — in a different way and with different techniques.

This book is a book of collaboration and research. It also defines new goals and presents a new understanding. It is not (yet) a textbook for graduate students. It rather informs our collaborators about a definite progress in the two above mentioned areas.

This research has been long; and at times hard and difficult. It has been a strain on our friends and companions. Thanks are due: Abbas Bahri wishes to thank Haim Brezis, his long-time collaborator and friend, not only for his obvious support but more so for his friendship. Having a friend — and of such a quality — is a rare blessing in life.

Abbas Bahri wishes also to thank Diana Nunziante, his wife, for her patience, her understanding and her love as this book was being written.

Lines and equations are written, but only with the overwhelming intelligence and love of those closest to us.

Both of us extend our warmest thanks to Barbara Mastrian for her wonderful work as well as her wit and life. It has been a pleasure to work with her all these years.

Finally, we would like to thank H. Brezis, S. Chanillo, R. Nussbaum and Z. Han for giving up so much of their time and patiently listen to our arguments as they developed.

We also thank them, as well as our friends and colleagues of the Mathematics Department at Rutgers, for their thoughtful remarks and observations.