Contents

Preface vii

1. Random Polymer Models and their Applications 1

1.1 Random Polymers and (De)Localization Phenomena 1
1.2 A First Model: Pinning on a Defect Line 6
1.2.1 Pinning a walk on a defect line 6
1.2.2 Pinning more general processes on a defect line . . . 13
1.3 Entropic Repulsion and Wetting Phenomena 17
1.3.1 Walls versus penetrable substrates 20
1.4 The Denaturation Transition: Poland–Scheraga Models . . 21
1.5 Force Induced Unzipping 26
1.6 Inhomogeneous Charge Distributions: Copolymers and Pinning 29
1.6.1 Periodic pinning models 30
1.6.2 Copolymers, periodic copolymers and selective interfaces 32
1.6.3 Copolymers with adsorption 36
1.7 Fully Inhomogeneous Charge Distributions and Disordered Polymer Models 37
1.7.1 Annealed models 41
1.8 On the Return Time Viewpoint 42
1.9 On Related Classes of Models 43
1.9.1 More general return times 43
1.9.2 Rewards and penalties on general structures 45
1.10 Bibliographic Complements 46

2. The Homogeneous Pinning Model 49
2.1 The Free Energy 49
2.2 Renewal Theory and Sharp Estimates 52
2.3 Path Properties: The Infinite Volume Limit 56
2.4 Path Properties: The Scaling Limit 57
2.5 Bibliographic Complements 63

3. Weakly Inhomogeneous Models 69
3.1 A Formula for the Free Energy: Reduction to a Finite Dimension Problem 69
3.2 Sharp Estimates on the Partition Function 75
3.3 The Limit Path Trajectories 77
3.3.1 The infinite volume limit 78
3.3.2 The scaling limit 80
3.4 First Order Transitions, Non-Uniqueness and Phase Diagrams 82
3.5 Bibliographic Complements 87

4. The Free Energy of Disordered Polymer Chains 89
4.1 Preliminary Observations 89
4.2 Existence of the Free Energy and Self-Averaging 90
4.3 Periodic versus Disordered Charge Sequences 94
4.4 Alternative Approaches to the Existence of the Free Energy 96
4.4.1 Concentration inequalities and self-averaging 96
4.4.2 The Super-Additive Ergodic Theorem approach . . . 97
4.5 Bibliographic Complements 98

5. Disordered Pinning Models: The Phase Diagram 101
5.1 Preliminary Observations on the Free Energy 101
5.2 An Improved Lower Bound on the Free Energy 102
5.3 On the Upper Bound: Annealing and Constrained Annealing 109
5.4 The Effect of the Disorder on the Order of the Transition: A Smoothing Inequality 111
5.5 What is One Expecting: The Renormalization Group Viewpoint and the Harris Criterion 116
5.6 A Fully Inhomogeneous Non-Disordered Model 118

5.7 Bibliographic Complements 122

6. Disordered Copolymers and Selective Interfaces: The Phase Diagram 127

6.1 Copolymers and Homogeneous Localization 130
6.2 Rare Stretch Strategy for Localization Estimates 135
6.3 The Weak Coupling Limit: Brownian Motion Model and Universality 140
6.3.1 On more general copolymer models and the Brownian scaling 142
6.3.2 Copolymers with adsorption 144
6.4 Bibliographic Complements 146

7. The Localized Phase of Disordered Polymers 151

7.1 A First Tightness Estimate 151
7.2 Quenched versus Quenched Averaged Estimates 154
7.3 Quenched Averaged Estimates and the Infinite Volume Polymer Measure 160
7.4 Bibliographic Complements 164

8. The Delocalized Phase of Disordered Polymers 167

8.1 Large Deviations and Delocalization 168
8.2 Beyond Large Deviations: $O(\log N)$ Results 170
8.3 The Strongly Delocalized Regime 174
8.4 Bibliographic Complements 177

9. Numerical Algorithms and Computations 181

9.1 Computing the Partition Function 182
9.1.1 Random walk models 182
9.1.2 General renewal models 183
9.1.3 The Fixman–Freire Algorithm 184
9.2 A Statistical Test for Localization 185
9.3 Applications: A Few Examples 188
9.4 Bibliographic Complements 191

Appendix A Mathematical Tools 193

A.1 Limit Theorems in Probability 193
A.1.1 Some remarks on convexity 196

A.2 Entropy 196
A.3 Concentration Properties 198
A.4 Slow Variation and Laplace Transforms 200
A.5 Renewal Theory for $\mathbb{N}$ Valued Random Variables 202
A.5.1 The renewal theorem 204
A.5.2 The renewal mass function: Transient case 206
A.5.3 The renewal mass function: Null recurrent case . . . 207
A.5.4 Scaling limits of renewal processes 208
A.6 Random Walks 212
A.7 The Super-Additive Ergodic Theorem 214
A.8 Perron–Frobenius Theory 216

Appendix B Some Technical Estimates 219

B.1 Homogeneous Localization Strategy: The Entropy Density 219
B.2 The Weak Localization Limit 221
B.3 Localization and Loss of Memory 222

Appendix C Effective Interface Models 229

C.1 Ising Model, Effective Interface Models and Random Walks 229

Bibliography 233

Index 241