

# Contents

<i>Preface</i>	vii
1. Introduction	1
2. Super algebras	7
2.1 The definition of a super algebra . . . . .	7
2.2 Homomorphisms and modules of super algebras . . . . .	9
2.3 Super matrices . . . . .	12
2.4 Super Lie algebras and super Lie modules . . . . .	13
3. Superspace	17
3.1 Real Grassmann algebras . . . . .	17
3.2 The topology of superspace . . . . .	21
3.3 Complex Grassmann algebras . . . . .	23
3.4 Further super matrices . . . . .	25
4. Functions of anticommuting variables	31
4.1 Superdifferentiation and finite-dimensional Grassmann algebras . . . . .	33
4.2 Taylor expansion and Grassmann analytic continuation . .	35
4.3 Supersmooth functions on $\mathbb{R}_S^{m,n}$ . . . . .	38
4.4 Properties of supersmooth functions . . . . .	40
4.5 Other infinite-dimensional algebras . . . . .	44
4.6 Obtaining well defined odd derivatives with finite- dimensional Grassmann algebras . . . . .	45
4.7 The inverse function theorem . . . . .	47

4.8	Partitions of unity . . . . .	49
4.9	Superholomorphic functions of complex Grassmann variables	50
5.	Supermanifolds: The concrete approach	51
5.1	$G^\infty$ DeWitt supermanifolds . . . . .	52
5.2	The topology of supermanifolds . . . . .	55
5.3	More general supermanifolds . . . . .	56
5.4	The body of a supermanifold . . . . .	59
5.5	Complex supermanifolds . . . . .	61
6.	Functions and vector fields	63
6.1	$G^\infty$ functions on supermanifolds . . . . .	64
6.2	Functions between supermanifolds . . . . .	67
6.3	Tangent vectors . . . . .	69
6.4	Vector fields . . . . .	74
6.5	Induced maps and integral curves . . . . .	78
7.	Supermanifolds: The algebro-geometric approach	85
7.1	Algebro-geometric supermanifolds . . . . .	85
7.2	Local coordinates on algebro-geometric supermanifolds . .	87
7.3	Maps between algebro-geometric supermanifolds . . . . .	89
8.	The structure of supermanifolds	91
8.1	The construction of a split supermanifold from a vector bundle	92
8.2	Batchelor's structure theorem for $(\mathbb{R}_S^{m,n}, \text{DeWitt}, G^\infty)$ supermanifolds . . . . .	94
8.3	A non-split complex supermanifold . . . . .	96
8.4	Comparison of the algebro-geometric and concrete approach . . . . .	98
9.	Super Lie groups	101
9.1	The definition of a super Lie group . . . . .	102
9.2	Examples of super Lie groups . . . . .	104
9.3	The construction of a super Lie group with given super Lie $\mathbb{R}_{S[L]}$ -module . . . . .	107
9.4	The super Lie groups which correspond to a given super Lie algebra . . . . .	114

9.5	Super Lie groups and the algebro-geometric approach to supermanifolds . . . . .	118
9.6	Super Lie group actions and the exponential map . . . . .	121
10.	Tensors and forms	125
10.1	Tensors . . . . .	125
10.2	Berezinian densities . . . . .	126
10.3	Exterior differential forms . . . . .	127
10.4	Super forms . . . . .	131
11.	Integration on supermanifolds	135
11.1	Integration with respect to anti commuting variables . . . . .	136
11.2	Integration on $\mathbb{R}_S^{m,n}$ . . . . .	140
11.3	Integration on compact supermanifolds . . . . .	144
11.4	Rothstein's theory of integration on non-compact supermanifolds . . . . .	146
11.5	Voronov's theory of integration of super forms . . . . .	152
11.6	Integration on $(1, 1)$ -dimensional supermanifolds . . . . .	154
11.7	Integration of exterior forms . . . . .	155
12.	Geometric structures on supermanifolds	157
12.1	Fibre bundles . . . . .	157
12.2	The frame bundle and tensors . . . . .	160
12.3	Riemannian structures . . . . .	161
12.4	Even symplectic structures . . . . .	162
12.5	Odd symplectic structures . . . . .	164
13.	Supermanifolds and supersymmetric theories	167
13.1	Superfields and the superspace formalism . . . . .	170
13.2	Supergravity . . . . .	175
13.3	Super embeddings . . . . .	178
14.	Super Riemann surfaces	181
14.1	The superspace geometry of the spinning string . . . . .	182
14.2	The definition of a super Riemann surface . . . . .	184
14.3	The supermoduli space of super Riemann surfaces . . . . .	186
14.4	Contour integration on super Riemann surfaces . . . . .	189
14.5	Fields on super Riemann surfaces . . . . .	191

15. Path integrals on supermanifolds	195
15.1 Path integrals and fermions . . . . .	195
15.2 Fermionic Brownian motion . . . . .	197
15.3 Brownian motion in superspace . . . . .	199
15.4 Stochastic calculus in superspace . . . . .	201
15.5 Brownian paths on supermanifolds . . . . .	203
16. Supermanifolds and BRST quantization	207
16.1 Symplectic reduction . . . . .	208
16.2 BRST cohomology . . . . .	211
16.3 BRST quantization . . . . .	213
16.4 A topological example . . . . .	215
17. Supermanifolds and geometry	221
17.1 Supermanifolds and differential forms . . . . .	222
17.2 Supermanifolds and spinors . . . . .	224
17.3 Supersymmetric quantum mechanics and the Atiyah Singer Index theorem . . . . .	226
17.4 Further applications of supermanifolds . . . . .	233
Appendix A. Notation	237
<i>Bibliography</i>	239
<i>Index</i>	249