Preface

These notes were written for a short series of lectures at the Scuola Normale Superiore, Pisa, in the academic years 1993-94, 1994-95, 1995-96, in order to fulfill a request by the mathematicians to have a short (half-semester) course for mathematical students willing to learn the main ideas and mathematical structure of Quantum Mechanics.

Since the books on Quantum Mechanics are hundreds and some of them very good, also for a mathematically minded student, one could reasonably question the need of having another one. The main point is that, contrary to Classical Mechanics, Quantum Mechanics is not part of the common education of mathematical students, even if this theory is at the roots of important developments of modern mathematics, as briefly discussed in the Introduction. Perhaps one of the reasons is that most of the books on Quantum Mechanics start from emphasizing and discussing the many experimental facts, which led to the crisis of Classical Mechanics, whose appreciation require a non-superficial familiarity with classical physics, with the result that the tight logical and mathematical structure of Quantum Mechanics is usually relegated to a secondary level. In particular, the usual presentation, which assumes that states are described by vectors of a Hilbert space (and observables by Hilbert space operators) on the basis of the so-called superposition principle, may not be appealing to a mathematical student. On the other hand, the books on the mathematical foundations of Quantum Mechanics, above all the beautiful books by J. Von Neumann, G. Mackey, J.M. Jauch, C. Piron, do not seem to have the introductory character suitable for undergraduate students.

Moreover, the C^*-algebraic formulation of Quantum Mechanics, which has unquestionable advantages for logic and conceptual economy, especially for a mathematically oriented audience, and has played a crucial role for the recent non-commutative extensions of Calculus, Geometry, Probability etc., has not yet become standard in quantum mechanics textbooks.

The aim of these notes is to introduce the mathematical student to the basic mathematical structure and foundations of Quantum Mechanics, by emphasizing as a starting point the C^*-algebraic structure of the algebra of observables, which defines a physical system.

An effort is made in motivating and possibly simplifying Segal's postulates for Quantum Mechanics; the C^*-algebraic structure of the algebra generated by the observables is argued to follow essentially from simple basic physical ideas, by exploiting the operational definition of measurements and the duality between states and observables.

Once the C^*-algebraic structure of the observables is accepted, the mathematical structure of Quantum Mechanics, namely the description of the physical states by Hilbert space vectors and the representation of the observables by Hilbert space operators, then follows from the GNS and Gelfand-Naimark theorems; Schroedinger Quantum Mechanics is then the direct consequence of the Heisenberg commutation relations and the Von Neumann uniqueness theorem.

No pretension of completeness is made about the subject; the scope is that of an introductory short course on the basic knowledge of Quantum Mechanics which should hopefully become part of the common education also of mathematicians. A few basic examples of quantum systems, *in primis* the hydrogen atom, are worked out with attention to the mathematical setting and logic.

An effort is made in simplifying the technical points and the proofs, especially in the short account of C^*-algebra theory, and in reducing the background knowledge needed to the minimum; the basic notions of probability theory and of Hilbert space operators are required.

The chapters or paragraphs marked with an asterisk were not part of the short course and were discussed in additional lectures; they are included here for completeness and may be skipped in a first reading. The increasing interest on the functional integral approach to quantum mechanics and more generally the deep connection between quantum mechanics and stochastic processes has motivated the brief account given in Chapter 6, with emphasis on the main ideas and the basic mathematical structures.

The effectiveness of the functional integral approach for both the solution of quantum mechanical problems and the analysis of general properties is briefly discussed in Chapter 6; in particular its relevance for Feynman perturbative expansion, the classical limit, the interplay between quantization and topology, the discussion of coupling constant analyticity are worked out in some simple, but illuminating cases.

Since the C^*-algebraic approach has proved inevitable for the mathematical description of quantum systems with infinite degrees of freedom, and useful for the discussion of deep theoretical physics problems, like spontaneous symmetry breaking, phase transitions, quantum statistical mechanics, quantum field theory etc., the lectures might turn out to be useful also for theoretical physics students, who might also profit from a look to the functional integral approach beyond the heuristic presentations usually adopted in theoretical physics books, especially in view of its use for a non-perturbative approach to quantum field theory and many body theory.

I am grateful to the *Accademia dei Lincei* and the University of Rome "*La Sapienza*" for inviting me to give a series of lectures and for the opportunity of confronting this presentation of Quantum Mechanics with a very selected audience. The preparation of these lectures notes has benefited from enlightening discussions and collaboration with G. Morchio, to whom I am greatly indebted. I thank also the students of my courses for pointing out misprints or unclear points in the preliminary draft of the notes.