Preface

This book is based on lecture notes used in a one-semester mathematical methods course for undergraduates at the University of Illinois at Chicago. The course is offered for students who have already taken an introductory physics course and serves as a prerequisite for more advanced courses in electricity and magnetism, classical mechanics, thermal and statistical physics, and quantum mechanics. For many students this course is their first encounter with advanced geometrical and algebraic concepts as well as with more challenging technical problems that require some degree of computational skills. A student's performance in the course is usually a very good indicator of how well he/she will do in advanced undergraduate physics and engineering courses.

The course is accompanied by a computer-based workshop, which guides students through the material using symbolic computation with Maple in the setting of a computer lab. Maple software was found to aid students to conceptualize the mathematical content of the course, and to encourage them to experiment with Mathematics.

A central feature of our book is that it teaches mathematical methods by integrating computer software with a traditional presentation of the material reflecting the pedagogical philosophy of the course. The advent of computers has changed the way people perform research, but, up until now, has had a relatively low impact on classroom instruction. Our book tries to change this status quo by using Maple as an integral component of the teaching process. However, we believe that Maple should not replace traditional mathematical theory (as calculators should not replace knowledge of the multiplication table). Therefore, as authors, we made a conscious decision to split our presentation of the book into two parallel parts: Part I contains an exposition of main topics in mathematical methods and Part II contains a computer-based approach to the same material.

The two parts of our book were designed to complement each other by introducing subject material via two different paths. The use of Maple allows us to omit several technical derivations and facts, thus, providing additional space for broadening the range of problems and applications of mathematical methods. The book contains a variety of problems chosen for their instructional value and physical relevance. Maple can effectively be used to assist students with tedious calculations that are required to solve the problems.

Part I essentially follows the old fashioned "paper and pencil" presentation of mathematical methods. It provides a streamlined and self-contained text of a mathematical methods course that emphasizes concepts that are important from the application perspective. Part I teaches a variety of technical tricks that are needed to successfully apply mathematical methods to solve important physical problems. The emphasis of Part I is on examples; some of the worked out examples illustrate how theorems work, others present their applications. By focusing on conceptual understanding of new ideas and geometrical intuition we attempted to write our book so as to reflect the way that most of physicists and engineers think about mathematics. The mathematical prerequisites are minimal (two semesters of calculus, including knowledge of the chain rule) and no pretense of mathematical rigor is made.

Part II of this book follows a computer-oriented approach to instruction. Topics in Part II are presented in the same order as in Part I. By closely mirroring part I, the second part reinforces it and improves its pedagogy. Maple is used in Part II both as a vehicle to teach mathematics and as a tool to solve mathematical problems. By studying Part II, students gain familiarity with a powerful computer algebra system that they will likely employ in other mathematics, engineering, or science courses. In Part II, Maple provides an advanced visualization environment that is sure to enhance students' grasp of traditional mathematical techniques and heighten their interest in the subject material.

There are a few possible ways in which the book may be used for instruction. One conventional option is a one-semester, four credit hours, undergraduate course covering the first five chapters (and their Maple counterparts, chapters 8-13), with possible omission of material students already know or parts that are too advanced. A much preferable option is a two-semester, three credit hours, undergraduate course which should cover the entire book from the beginning till the end.

Part I, contains seven chapters that cover most relevant topics, such as vector calculus, matrices and the eigenvalue problem, differential equations, power series solutions, Frobenius theorem, orthogonal eigenfunction expansion, Fourier series and the stability of the differential linear systems. The seventh, and final, chapter introduces students to fundamental concepts of nonlinear differential equations. It introduces contemporary and fundamental topics like nonlinear differential equations, chaos and solitons.

Throughout the book we teach students to think about functions as elements of a vector space. An abstract formulation of a vector space is essential for a complete understanding of the Fourier series and, more generally, the Sturm-Liouville problems with orthogonal eigenfunction expansions. This universal way of thinking about various forms of eigenfunctions expansions is especially useful for those students who will go on to take a quantum mechanics course. We introduce an abstract notion of a vector space early in the text and illustrate it by examples of function and polynomial vector spaces. Similarly, the Gram-Schmidt procedure, initially introduced in the setting of three-dimensional space, is extended to the n-dimensional vector space and to the infinite- dimensional vector space of orthogonal eigenfunctions.

The concept of orthogonal curvilinear coordinates is core to the mathematical methods curriculum. A discussion of orthogonal curvilinear coordinates is given in Chapter 1, in considerable detail, and precedes the subject of surface integrals since it can be used in calculation of many surface integrals, which can conveniently be formulated in terms of spherical or cylindrical coordinates.

Complex numbers and functions appear frequently in the text. They are explained in the Appendix A. We have omitted in this project the subject of residue calculus, a traditional goal of chapters on complex variables. Our decision was based on the fact that Maple easily calculates the relevant integrals.

This book was written with LaTeX . With a few exceptions, figures were produced either by Maple or PSTricks, a set of powerful macros for drawing high-quality graphs that can be included in TeX or LaTeX documents. We are indebted to a very active PSTricks community for making many wonderful macro packages and examples available for public use.

We should say at the outset that there is nothing in this book which is truly original. Although several of the logical connections and pedagogical arguments were invented in the process of writing this book, the very conventional character of the material makes it neither possible to claim originality nor to fully acknowledge our predecessors as generously we would like to. It is impossible, when writing a book of this nature, to give credit where it belongs because of the vast amount of pre-existing material. We profoundly apologize for not quoting countless references and sources, which are available. The only exception is the note after Chapter 7, which lists a few selected sources that influenced us most in the process of writing.

We are very grateful to Yvonne Aratyn for her patient help with editing large parts of Part I of the book and correcting numerous instances of confusing passages and bad grammar. We are indebted to students in Physics 215 classes at the University of Illinois at Chicago for reporting many typos and errors.

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