CHAPTER 1

Introduction

1.1 Books

W. Pauli, "*Die Allgemeinen Principien der Wellenmechanik*"; *Handbuch der Physik*, 2 ed., Vol. 24,

Part 1; Edwards reprint, Ann Arbor 1947. (In German) [1]

W. Heitler, *Quantum Theory of Radiation*, 2nd Edition, Oxford. 3rd edition just published. [2]

G. Wentzel, *Introduction to the Quantum Theory of Wave-Fields*, Interscience, N.Y. 1949 [3]

I shall not expect you to have read any of these, but I shall refer to them as we go along. The later part of the course will be new stuff, taken from papers of Feynman and Schwinger mainly. [4], [5], [6], [7], [8]

1.2 Subject Matter

You have had a complete course in non-relativistic quantum theory. I assume this known. All the general principles of the non-relativistic theory are valid and true under all circumstances, in particular also when the system happens to be relativistic. What you have learned is therefore still good.

You have had a course in classical mechanics and electrodynamics including special relativity. You know what is meant by a system being relativistic; the equations of motion are formally invariant under Lorentz transformations. General relativity we shall not touch.

This course will be concerned with the development of a *Lorentz–invariant quantum theory.* That is not a general dynamical method like the non-relativistic quantum theory, applicable to all systems. We cannot yet devise a general method of that kind, and it is probably impossible. Instead we have to find out what are the possible systems, the particular equations of motion, which can be handled by the non-relativistic quantum dynamics and which are at the same time Lorentz–invariant.

In the non-relativistic theory it was found that almost any classical system could be handled, i.e. quantized. Now on the contrary we find there are very few possibilities for a relativistic quantized system. This is a most important fact. It means that starting only from the principles of relativity and quantization, it is mathematically possible only for very special types of objects to exist. So one can *predict* mathematically some important things about the real world. The most striking examples of this are:

(i) Dirac from a study of the electron predicted the positron, which was later discovered [9].
(ii) Yukawa from a study of nuclear forces predicted the meson, which was later discovered [10].

These two examples are special cases of the general principle, which is the basic success of the relativistic quantum theory, that *A Relativistic Quantum Theory of a Finite Number of Particles is Impossible.* A relativistic quantum theory necessarily contains these features: an indefinite number of particles of one or more types, particles of each type being identical and indistinguishable from each other, possibility of creation and annihilation of particles.

Thus the two principles of relativity and quantum theory when combined lead to a world built up out of various types of elementary particles, and so make us feel quite confident that we are on the right way to an understanding of the real world. In addition, various detailed properties of the observed particles are necessary consequences of the general theory. These are for example:

(i) Magnetic moment of Electron (Dirac) [9].
(ii) Relation between spin and statistics (Pauli) [11].

1.3 Detailed Program

We shall not develop straightaway a correct theory including many particles. Instead we follow the historical development. We try to make a relativistic quantum theory of *one* particle, find out how far we can go and where we get into trouble. Then we shall see how to change the theory and get over the trouble by introducing many particles. Incidentally, the one–particle theories are quite useful, being correct to a good approximation in many situations where creation of new particles does not occur, and where something better than a non-relativistic approximation is needed. An example is the Dirac theory of the Hydrogen atom.[1]

The non-relativistic theory gave levels correctly but no fine-structure. (Accuracy of one part in 10,000). The Dirac one-particle theory gives all the main features of the fine-structure correctly, number of components and separations good to 10% but not better. (Accuracy one part in 100,000).

The Dirac many-particle theory gives the fine-structure separations (Lamb experiment) correctly to about one part in 10,000. (Overall accuracy 1 in 10^8.)

Probably to get accuracy better than 1 in 10^8 even the Dirac many-particle theory is not enough and one will need to take all kinds of meson effects into account which are not yet treated properly. Experiments are so far only good to about 1 in 10^8.

In this course I will go through the one-particle theories first in detail. Then I will talk about their breaking down. At that point I will make a fresh start and discuss how one can make a relativistic quantum theory in general, using the new methods of Feynman and Schwinger. From this we shall be led to the many-particle theories. I will talk about the general features of these theories. Then I will take the special example of quantum electrodynamics and get as far as I can with it before the end of the course.

1.4 One-Particle Theories

Take the simplest case, one particle with no forces. Then the non-relativistic wave-mechanics tells you to take the equation $E = \frac{1}{2m}p^2$ of classical mechanics, and write

$$E \to i\hbar\frac{\partial}{\partial t} \qquad p_x \to -i\hbar\frac{\partial}{\partial x} \tag{1}$$

to get the wave-equation[2]

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi = -\frac{\hbar^2}{2m}\nabla^2\psi \tag{2}$$

satisfied by the wave-function ψ.

To give a physical meaning to ψ, we state that $\rho = \psi^*\psi$ is the probability of finding the particle at the point x, y, z at time t. And the probability is conserved because[3]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{\boldsymbol{\jmath}} = 0 \tag{3}$$

where

$$\vec{\boldsymbol{\jmath}} = \frac{\hbar}{2mi}(\psi^* \nabla \psi - \psi \nabla \psi^*) \tag{4}$$

where ψ^* is the complex conjugate of ψ.

Now do this relativistically. We have classically

$$E^2 = m^2 c^4 + c^2 p^2 \tag{5}$$

which gives the wave equation

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi = \nabla^2 \psi - \frac{m^2 c^2}{\hbar^2}\psi\,. \tag{6}$$

This is an historic equation, the Klein–Gordon equation. Schrödinger already in 1926 tried to make a relativistic quantum theory out of it. But he failed, and many other people too, until Pauli and Weisskopf gave the many-particle theory in 1934 [12]. Why?

Because in order to interpret the wave-function as a probability we must have a continuity equation. This can only be got out of the wave-equation if we take $\vec{\boldsymbol{\jmath}}$ as before, and

$$\rho = \frac{i\hbar}{2mc^2}\left(\psi^* \frac{\partial \psi}{\partial t} - \frac{\partial \psi^*}{\partial t}\psi\right)\,. \tag{7}$$

But now since the equation is second order, ψ and $\frac{\partial \psi}{\partial t}$ are arbitrary. Hence ρ need not be positive. We have *negative probabilities.* This defeated all attempts to make a sensible one-particle theory.

The theory can be carried through quite easily, if we make ψ describe an assembly of particles of both positive and negative charge, and ρ is the *net* charge density at any point. This is what Pauli and Weisskopf did, and the theory you get is correct for π-mesons, the mesons which are made in the synchrotron downstairs. I will talk about it later.