Preface

Group theory is a powerful tool for studying the symmetry of a physical system, especially the symmetry of a quantum system. Since the exact solution of the dynamic equation in the quantum theory is generally difficult to obtain, one has to find other methods to analyze the property of the system. Group theory provides an effective method by analyzing the symmetry of the system to obtain some precise information of the system verifiable with observations. Now, Group Theory is a required course for graduate students majored in physics and theoretical chemistry.

The course of Group Theory for the students majored in physics is very different from the same course for those majored in mathematics. A graduate student in physics needs to know the theoretical framework of group theory and more importantly to master the techniques in the application of group theory to various fields of physics, which is actually his main objective for taking the course. However, no course or textbook on group theory can be expected to include explicitly the solution to every problem of group theory in his research field of physics. A student of physics has to know the fundamental theory of group theory, otherwise he may not be able to apply the techniques creatively. On the other hand, the student of physics is not expected to completely grasp all the mathematics behind group theory due to the breadth of the knowledge required.

I first taught the group theory course in 1962. Since 1986, I have been teaching for 20 years the course of Group Theory to graduate students mainly majored in physics at the Graduate School of Chinese Academy of Sciences. In addition, most of my research work has been related to applications of group theory to physics. In 1996, the Chinese Academy of Sciences decided to publish a series of textbooks for graduate students. I was invited to write a textbook on Group Theory for the series. In the textbook, based on my experience in teaching and research work, I explained the fundamental concepts and techniques of group theory progressively and systematically using the language familiar to physicists, and also emphasized the ways with which group theory is applied to physics. The textbook (in Chinese) has been widely used for the Group Theory course in China since it was published by Science Press (Beijing) in 1998. The second edition of this textbook was published in 2006 after systematic revision. Up to the seventh printing in December 2006, 16800 copies of this textbook were printed altogether. This textbook is the foundation for the present textbook, although some new materials are included and some are moved to another exercise book [Ma and Gu (2004)].

By the request of the readers, an exercise book on group theory, by the same author, was published in 2002 by Science Press (Beijing) to form a complete set of textbooks on group theory. In order to make the exercise book self-contained, a brief review of the main concepts and techniques is given before the problems in each section. The reviews can be used as a concise textbook on group theory. Cooperated with my student, the English version of the exercise book was published in 2004 by World Scientific named "Problems and Solutions in Group Theory for Physicists" ([Ma and Gu (2004)]). A great deal of new materials drawn from teaching and research experiences is included. The reviews of each chapter has been extensively revised. Last four chapters are essentially new. Some useful results are listed in the exercise book for reference. Some materials are moved from the textbook to the exercise book such that the textbook can concentrate the main subjects on group theory and include some new developments in group theory. This exercise book serves as a supplement for the present textbook.

The present textbook consists of 10 chapters. Chapter 1 is a short review on linear algebra. The reader is required not only to be familiar with its basic concepts but also to master its applications, especially the similarity transformation method. In Chap. 2, the concepts of a group and its subsets are introduced from the physical problems and explained through examples of some finite groups. The importance of the group table of a finite group is emphasized. The group table of the symmetric groups of regular polyhedrons are introduced in a new way (see §2.5.2). The theory of representations of a group is studied in Chap. 3. The transformation operator P_R for the scalar functions bridges the gap between the representation theory and physical application. The subduced and induced representations of groups are used to construct the character tables of finite groups in

Chap. 3, and to calculate the outer product of representations of the permutation groups in Chap. 6. The method of idempotents is systematically studied in §3.7. Based on this method the standard irreducible basis vectors in the group algebras of some finite groups in common use in physics are calculated. The method of Young operators studied in Chap. 6 is its development to the permutation groups.

The classification and representations of semisimple Lie algebras are introduced in Chap. 7 and partly in Chap. 4 by the language familiar to physicists. The methods of block weight diagrams and dominant weight diagrams are recommended for calculating the representation matrices of the generators and the Clebsch−Gordan series in a simple Lie algebra. The readers who are interested in the strict mathematical definitions and proofs in the theory of semisimple Lie algebras are recommended to read the more mathematically oriented books (e.g. [Bourbaki (1989)]).

The remaining part of the book is devoted to the important symmetric groups of physical systems. In Chap. 4 the symmetric group SO(3) of a spherically symmetric system in three dimensions is studied. The study on the symmetry of crystals is a typical example of the physical application of group theory. In Chap. 5, through a systematic analysis by the method of group theory, only based on the translation symmetry of crystals, the crystals are classified completely that there are 11 proper crystallographic point groups, 32 crystallographic point groups, 7 crystal systems, 14 Bravais lattices, 73 symmorphic space groups, and 230 space groups. The international symbols of the space groups are recommended in Chap. 5. The analysis method for the symmetry of a crystal from its International symbols are emphasized (see §5.4.6). The permutation groups S_n are the symmetric groups of the identical particles and are widely used in the decomposition of the tensor spaces. In Chap. 6 the permutation groups S_n are studied by the method of Young operators. The irreducible basis vectors in the group space of S_n are explicitly given based on the Young operators, and the calculating method for the similarity transformations from them to the orthonormal basis vectors is proved and demonstrated by examples. The matrix groups in common use in physics, such as the SU(N) groups, the SO(N) groups, the USp(2ℓ) groups, and the Lorentz group L_p are studied in some detail in the last three chapters.

The systematic examination of Young operators is an important characteristic of this book. We calculate the characters, the representation matrices, and the outer product of the irreducible representations of the permutation groups using the method of Young operators. For the matrix groups SU(N), SO(N), and USp(2ℓ), which are related to four classical Lie algebras, the basis states of the irreducible representations can be explicitly calculated using the method of Young operators. The dimensions of the irreducible representations of the permutation groups, the SU(N) groups, the SO(N) groups, and the Sp(2ℓ) groups are all calculated by the hook rule, a method based on the Young patterns.

An isolated quantum n-body system is invariant in the translation of space−time and the spatial rotation so that the energy, the momentum, and the total angular momentum of the system are conserved. The motion of the center-of-mass and the global rotation of the system should be separated from its internal motion, and its Schrödinger equation is reduced to the radial equation, depending only on the internal degrees of freedom. The method of the Jacobi coordinate vectors is summarized in §4.9.1 to separate the motion of the center-of-mass. The generalized harmonic polynomials are presented to separate the global rotation of an isolated quantum n-body system from the internal motion in §4.9 and generalized to arbitrary N-dimensional space in §9.4, where the Dirac equation in $(N+1)$-dimensional space−time is also studied.

In conclusion, I must express my cordial gratitude to my supervisor Prof. Ning Hu for his guidance, under which I entered the research field of the symmetric theory of particle physics. I am indebted to Prof. Yi-Shi Duan who made me an abecedarian to study and to teach the group theory. I am very grateful to my wife, Ms Xian Li, for her continuous support. This book was supported by the National Natural Science Foundation of China under Grant Nos. 10475082 and 10675050.

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Beijing, China
June 2007

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