

Contents

Preface

vii

Part I: Fundamental Concepts

Chapter 1

Bipartite and Multipartite Entanglement of Gaussian States 1

G. Adesso and F. Illuminati

1	Introduction	2
2	Gaussian States of Continuous Variable Systems	2
3	Two-Mode Gaussian States: Entanglement and Mixedness	4
4	Multimode Gaussian States: Unitarily Localizable Entanglement	9
5	Entanglement Sharing of Gaussian States	13
6	Exploiting Multipartite Entanglement: Optimal Fidelity of Continuous Variable Teleportation	16
7	Conclusions and Outlook	19
	References	20

Chapter 2

Gaussian Quantum Channels 23

J. Eisert and M. M. Wolf

1	Introduction	23
2	Gaussian Channels	24
	2.1 Preliminaries	24
	2.2 General Gaussian channels	25
	2.3 Important examples of Gaussian channels	27
3	Entropies and Quantum Mutual Information	28
	3.1 Output entropies	28
	3.2 Mutual information and coherent information	29
	3.3 Entropies of Gaussian states and extremal properties	30
	3.4 Constrained quantities	30
4	Capacities	31
	4.1 Classical information capacity	32
	4.2 Quantum capacities and coherent information	34
	4.3 Entanglement-assisted capacities	35
5	Additivity Issues	37
	5.1 Equivalence of additivity problems	37
	5.2 Gaussian inputs to Gaussian channels	38

5.3 Integer output entropies and Gaussian inputs	39
6 Outlook	39
References	40

Chapter 3

Entanglement in Systems of Interacting Harmonic Oscillators

43

K. M. R. Audenaert, J. Eisert and M. B. Plenio

1 Introduction	43
2 Systems of Harmonic Oscillators	44
3 Static Properties of Harmonic Chains	47
4 Dynamical Properties of Harmonic Chains	55
5 Summary and Conclusions	61
References	61

Chapter 4

Continuous-Variable Quantum Key Distribution

63

F. Grosshans, A. Acín and N. J. Cerf

1 Introduction	63
2 Generic Description of Continuous-Variable Protocols	64
3 Structure of the Security Proofs	67
3.1 Eve's physical attack	67
3.2 Eve's measurement	67
3.3 Eve's knowledge	68
4 Individual Attacks	69
4.1 Preliminaries	69
4.2 Secure key rates against individual attacks	71
5 Collective Attacks	72
5.1 Preliminaries	72
5.2 Secure key rates against collective attacks	74
6 Coherent Attacks	76
7 Optimality of Gaussian Attacks	78
7.1 Preliminaries	78
7.2 Entropy of Gaussian states $\tilde{\rho}$ — general attacks	79
7.3 Conditional entropy of $\tilde{\rho}$ — individual attacks	80
7.4 Effect of Alice's measurement — collective attacks	81
8 Conclusion	82
References	82

Chapter 5

Gaussian Quantum Cellular Automata

85

O. Krüger and R. F. Werner

1 Introduction	85
2 Classical Cellular Automata	87
3 Going Quantum	88
4 Gaussian Quantum Cellular Automata	90

5 Summary	98
References	99

Chapter 6

Distillation of Continuous-Variable Entanglement 101

J. Fiurášek, L. Mišta and R. Filip

1 Introduction	101
2 Entanglement Distillation of Gaussian States with Gaussian Operations is Impossible	102
3 Entanglement Concentration Based on Cross-Kerr Effect	108
4 Entanglement Concentration by Subtraction of Photons	112
5 Gaussification by Means of LOCC Operations	117
6 Conclusions	119
References	120

Chapter 7

Loophole-Free Test of Quantum Nonlocality with Continuous Variables of Light 121

R. García-Patrón, J. Fiurášek and N. J. Cerf

1 Introduction	121
2 Bell Inequalities	122
3 Experimental Bell Test and Related Loopholes	123
4 Bell Test with Continuous Variables of Light	124
5 Loophole-Free Bell Test Using Homodyne Detectors	127
6 Simplified Model with Ideal Photodetectors	129
7 Realistic Model	130
7.1 Calculation of the Wigner function	131
7.2 Resulting Bell violation	132
7.3 Sensitivity to experimental imperfections	134
8 Alternative Schemes	135
9 Conclusions	137
References	138

Chapter 8

Homodyne Tomography and the Reconstruction of Quantum States of Light 141

G. M. D'Ariano, L. Maccone and M. F. Sacchi

1 Introduction	141
2 Homodyne Tomography	143
2.1 Homodyne detection	144
2.2 Noise deconvolution	145
2.3 Adaptive tomography	146
3 Monte Carlo Methods for Tomography	146
4 Maximum Likelihood Tomography	148
5 Tomography for Dummies	150
6 Quantum Calibration of Measurement Devices	151

7 History of Quantum Tomography	156
References	157

Chapter 9

Schrödinger Cat States for Quantum Information

Processing

159

H. Jeong and T. C. Ralph

1 Introduction	159
2 Quantum Information Processing with Schrödinger Cat States	160
2.1 Coherent-state qubits	160
2.2 Quantum teleportation	161
2.3 Quantum computation	163
2.4 Entanglement purification for Bell-cat states	166
3 Production of Schrödinger Cat States	170
3.1 Schemes using linear optics elements	170
3.2 Schemes using cavity quantum electrodynamics	173
3.3 Schemes using weak nonlinearity	174
4 Conclusion	176
References	177

Part II: Optical Continuous Variables

Chapter 10

Polarization Squeezing and Entanglement

181

N. Korolkova

1 Introduction	181
2 Polarization Squeezing	182
3 Continuous Variable Polarization Entanglement	186
References	195

Chapter 11

Type-II Optical Parametric Oscillator:

A Versatile Source of Quantum Correlations and

Entanglement

197

J. Laurat, T. Coudreau and C. Fabre

1 Introduction	197
2 Correlation Criteria	198
2.1 “Gemellity”	198
2.2 Quantum Non Demolition correlation	199
2.3 Inseparability	200
2.4 Einstein-Podolsky-Rosen correlations	201
3 Experimental Investigation of Quantum Correlations	201
3.1 Experimental set-up	202
3.2 “ 2×1 quadrature” case	203

3.2.1	Twin beams	203
3.2.2	QND correlations and conditional preparation of a non-classical state	204
3.3	“ 2×2 ” quadratures case	206
3.3.1	Entanglement below threshold	206
3.3.2	Bright EPR beams above threshold and polarization squeezing	208
4	Manipulating Entanglement with Polarization Elements	210
4.1	Manipulation of entanglement in the two-mode state produced by the type-II OPO with mode coupling	210
4.2	Experimental optimization of entanglement	212
5	Conclusion	213
	References	213

Chapter 12

Accessing the Phase Quadrature of Intense Non-Classical Light State 215

O. Glöckl, U. L. Andersen and G. Leuchs

1	Introduction	215
2	Sideband Picture	216
3	Phase Measuring Interferometer — Principle of Operation	219
4	Phase Measuring Interferometer — Setup and Efficiency	222
5	Generation of Quadrature Entanglement	223
6	Different Phase Measurements	224
6.1	Phase modulated laser beam	224
6.2	Phase noise measurements of intense, short, amplitude squeezed pulses from a fibre	225
6.3	Sub-shot noise phase quadrature measurements	227
7	Conclusions	230
	References	230

Chapter 13

Experimental Polarization Squeezing and Continuous Variable Entanglement via the Optical Kerr Effect 233

V. Josse, A. Dantan, A. Bramati, M. Pinard, E. Giacobino, J. Heersink, U. L. Andersen, O. Glöckl and G. Leuchs

1	Introduction	233
2	Polarization Squeezing	235
2.1	Definition	235
2.2	Connection to vacuum squeezing	236
2.3	Generation of polarization squeezing: an example	237
3	Polarization Squeezing via Kerr Effect	238
3.1	The optical Kerr effect	238
3.2	Polarization squeezing with cold atoms	239
3.2.1	Nonlinear atom-light interaction in an optical cavity	239
3.2.2	Principle of polarization squeezing generation	239

3.2.3	Experimental setup	241
3.2.4	Results	242
3.3	Polarization squeezing with optical fibers	243
3.3.1	Nonlinear interaction of light in a glass fiber	243
3.3.2	Generation of polarization squeezing	244
3.3.3	Experimental setup	245
3.3.4	Results	246
4	Polarization and Quadrature Entanglement	247
4.1	General properties of continuous variable entanglement	248
4.1.1	Probing and quantifying entanglement	248
4.1.2	Finding maximum entanglement in a two mode system	249
4.1.3	Application and representation in the Poincaré sphere	251
4.2	Entanglement generation with cold atoms	252
4.2.1	Principle	252
4.2.2	Measurement and results	254
4.3	Entanglement generation from fibers	255
4.3.1	Direct generation of quadrature entanglement	255
4.3.2	Polarization entanglement: method and results	257
5	Conclusions and Outlook	259
	References	259

Chapter 14

High-Fidelity Quantum Teleportation and a Quantum Teleportation Network

265

N. Takei, H. Yonezawa, T. Aoki and A. Furusawa

1	Introduction	265
2	Quantum Teleportation	266
2.1	Teleportation of a coherent state	269
2.2	Teleportation of a squeezed state	271
2.3	Entanglement swapping	274
3	Quantum Teleportation Network	277
4	Conclusion and Outlook	282
	References	283

Chapter 15

Quantum State Sharing with Continuous Variables

285

T. Tyc, B. C. Sanders, T. Symul, W. P. Bowen, A. Lance and P. K. Lam

1	Introduction	285
2	Classical Secret Sharing	287
3	Quantum State Sharing with Discrete Variables	288
4	Quantum State Sharing with Continuous Variables	289
4.1	Linear mode transformations	290
5	The $(k, 2k - 1)$ CV Quantum State Sharing Threshold Scheme	291
6	The $(2, 3)$ Threshold Scheme	292
6.1	Encoding the secret state	292
6.2	Extraction of the secret state by players 1 and 2	293

6.3	Extraction of the secret state by players 1 and 3	294
6.3.1	Phase insensitive amplifier protocol	295
6.3.2	Two optical parametric amplifier protocol	295
6.3.3	Single feed-forward extraction protocol	296
6.3.4	Double feed-forward extraction protocol	296
7	Characterization of the Extraction Quality	297
7.1	Fidelity	297
7.2	Signal transfer and added noise	298
8	Experimental Realization of the (2, 3) Threshold Scheme	299
9	Conclusion	301
	References	302

Chapter 16

Experimental Quantum Cloning with Continuous Variables **305**

U. L. Andersen, V. Josse, N. Lütkenhaus and G. Leuchs

1	Introduction	305
2	Theory	306
2.1	Classical cloning	307
2.2	Quantum cloning	307
2.2.1	Previous proposals	307
2.2.2	Our proposal	309
2.3	Robustness	312
3	Experiment	313
3.1	Preparation	314
3.2	Cloning	314
3.3	Verification	314
3.4	Results	315
4	Non-Unity Gain Cloning	319
5	Other Cloning Functions	320
6	Conclusion	320
	References	321

Chapter 17

Quantum Imaging Techniques for Improving Information Extraction from Images **323**

C. Fabre, N. Treps, H. A. Bachor and P. K. Lam

1	Quantum Imaging: An Example of Multimode Quantum Optics	323
2	Quantum Imaging Using Many Modes	324
2.1	Generation of local quantum effects	325
2.2	Improvement of optical resolution	326
3	Quantum Imaging Using a Few Modes	327
3.1	Information extraction from images	327
3.2	Determination of the eigenmodes of the measurement	328
3.3	Case of beam nano-positioning	330
4	Synthesizing a Few-Mode Quantum State for Sub-Shot Noise Beam Nano-Positioning	332

4.1	1D nano-positioning	332
4.2	2D nano-positioning	334
4.3	Optimum detection of a beam displacement	337
4.4	Tilt and displacement measurement	339
5	Future Prospects	340
	References	342

Chapter 18

Squeezed Light for Gravitational Wave Detectors 345

R. Schnabel

1	Introduction	345
2	Quadrature Field Amplitudes in Frequency Space	348
3	Quantum Noise in Interferometers	351
3.1	Shot noise	351
3.2	Radiation pressure noise	352
3.3	Total quantum noise and the standard quantum limit	354
3.4	Quantum non-demolition interferometers	354
3.5	The dual-recycled Michelson interferometer	356
4	Generation of Squeezed States of Light	357
4.1	Squeezing from optical parametric oscillation and amplification	358
4.2	Squeezing at audio-band sideband frequencies	359
4.3	Frequency dependent squeezing	361
5	Towards Squeezing Enhanced Gravitational Wave Detectors	362
5.1	Table-top experiments	362
5.2	Outlook	364
6	Summary	364
	References	365

Chapter 19

Continuous Variables for Single Photons 367

L. Zhang, E. Mukamel, I. A. Walmsley, Ch. Silberhorn,

A. B. U'Ren and K. Banaszek

1	Introduction	368
2	Space-Time Structure of Single Photons	370
2.1	Measuring the space-time structure of photons	370
2.2	Measuring the joint space-time structure of photon pairs	372
2.2.1	Test of the EPR-paradox using photon pairs	372
2.2.2	Continuous-variable Bell inequality for photon pairs	374
3	Conditional Preparation of Pure-State Single Photons	376
3.1	Conditional preparation of single photons relying on PDC photon pairs	376
3.2	Factorization of the wave function in signal and idler modes for bulk crystals	378
4	Applications of Continuous Variables in Single Photons	380
4.1	Qudit information coding	380

4.2	Quantum key distribution with continuous variables and photon pairs	382
5	Conclusions and Outlook	384
	References	385

Chapter 20

Experimental Non-Gaussian Manipulation of Continuous Variables **389**

J. Wenger, A. Ourjoumtsev, J. Laurat, R. Tualle-Brouiri and P. Grangier

1	Introduction	389
2	Squeezed Vacuum Degaussification: A Theoretical Approach	392
3	Experimental Implementation	394
	3.1 Pulsed squeezed vacuum generation	396
	3.2 Time-resolved homodyne detection of pulsed squeezed vacuum	398
	3.3 De-Gaussification apparatus	399
4	Characterization of the Non-Gaussian States	400
	4.1 Homodyne measurements and influence of experimental imperfections	400
	4.2 Quantum tomography of the non-Gaussian states	402
5	Conclusion and Potential Applications of Non-Gaussian States	404
	References	406

Chapter 21

Continuous-Variable Quantum-State Tomography of Optical Fields and Photons **409**

A. I. Lvovsky and M. G. Raymer

1	Introduction	409
2	The Principles of Homodyne Tomography	412
	2.1 Inverse linear transform state reconstruction	413
	2.1.1 Wigner function	413
	2.1.2 Inverse Radon transformation	414
	2.2 Maximum-likelihood reconstruction	415
3	Homodyne Tomography of Discrete-Variable States	418
	3.1 Motivation	418
	3.2 Time-domain homodyne detection	419
	3.3 Matching the mode of the local oscillator	421
	3.4 Tomography of photons and qubits	425
	3.4.1 Single-photon Fock state	425
	3.4.2 Tomography of the qubit	427
	3.4.3 Nonlocality of the single photon and its consequences	428
	References	430

Part III: Atomic Continuous Variables

Chapter 22

Gaussian Description of Continuous Measurements on Continuous Variable Quantum Systems 435

L. B. Madsen and K. Mølmer

1	Introduction	435
2	Time Evolution of Gaussian States, General Theory	438
2.1	Time evolution due to a bilinear Hamiltonian	438
2.2	Time evolution due to dissipation and noise	438
2.3	Time evolution due to a homodyne measurement event	439
2.4	Time evolution due to continuous homodyne measurements	441
3	Application of the Gaussian Formalism to Atom-Light Interaction	443
3.1	Stokes vector and canonical conjugate variables for light	443
3.2	Atom-light interaction	444
3.2.1	Spin 1/2-case	445
4	Spin Squeezing in the Gaussian Description	447
4.1	Dissipation and noise	448
4.2	Solution of Ricatti equation	448
4.3	Inhomogeneous coupling	450
5	Magnetometry in the Gaussian Description	451
6	Entanglement in the Gaussian Description	454
6.1	Entanglement and vector magnetometry	455
7	Extensions of the Theory	455
7.1	Non spin-1/2 systems	456
7.2	Quantum correlated light beams	457
7.3	Beyond the Gaussian approximation	458
8	Conclusions and Outlook	459
	References	460

Chapter 23

Quantum State Preparation of Spin Ensembles by Continuous Measurement and Feedback 463

R. van Handel, J. K. Stockton, H. Mabuchi and H. M. Wiseman

1	Introduction	463
2	The Physical Model: From QED to Stochastic Equations	465
2.1	System model from quantum electrodynamics	465
2.2	Example: spins with dispersive coupling	469
3	Conditioning	470
3.1	Optical detection	470
3.2	The quantum filter	471
3.3	Conditional spin dynamics	472
4	Quantum Feedback Control	474
4.1	Separation structure	474

4.2	Defining an objective	476
4.3	Robustness and model reduction	477
5	Feedback in Atomic Ensembles	478
5.1	Spin squeezing in one ensemble	478
5.2	Dicke state preparation in one ensemble	480
5.3	Spin squeezing across two ensembles	482
6	Conclusion	483
	References	483

Chapter 24

Real-Time Quantum Feedback Control with Cold Alkali

Atoms **487**

J. M. Geremia

1	Introduction	487
2	The Atomic Spin System	489
2.1	Generating spin-squeezing using measurement	491
3	Continuous Measurement of Spin Angular Momentum	492
3.1	Continuous measurement as a scattering process	493
3.1.1	Physical interpretation	494
3.1.2	Irreducible representation of the scattering Hamiltonian	495
3.1.3	Scattering time-evolution operator	496
3.2	The continuous photocurrent	497
3.3	Physical interpretation of the photocurrent	498
4	Spin Squeezing	499
4.1	Filtering and the degree of squeezing	500
4.2	Real-time feedback control	501
5	Deterministic Spin-Squeezing Experiment	502
5.1	Experimental characterization of spin-squeezing	503
5.2	Squeezing data	504
5.3	Absolute spin-squeezing calibration	507
6	Conclusion	509
	References	509

Chapter 25

Deterministic Quantum Interface between Light and Atomic Ensembles

513

K. Hammerer, J. Sherson, B. Julsgaard, J. I. Cirac and E. S. Polzik

1	Introduction	513
2	Off-Resonant Interaction of Pulsed Laser Light with Spin Polarized Atomic Vapor	514
3	Equations of Motion	518
3.1	Single sample without magnetic field	518
3.2	Two samples in oppositely oriented magnetic fields	519
3.3	Single sample in magnetic field	520
4	The Role of Dissipation	522
5	Experimental Implementations	523

5.1	Determination of the projection noise level	524
5.2	The effect of atomic motion	525
5.3	Predicting the projection noise level	527
5.4	Thermal spin noise	529
5.5	Quantumness of the noise	530
6	Entanglement Generation and Verification	532
6.1	Theoretical entanglement modeling	533
6.2	Entanglement model with decoherence	534
6.3	Experimental entanglement results	535
7	Quantum Memory	536
7.1	Experimental verification of quantum memory	538
7.2	Decoherence	540
7.3	Quantum memory retrieval	541
8	Quantum Teleportation	541
8.1	Basic protocol	541
9	Multipass Interface	546
10	Prospects	549
	References	550

Chapter 26

Long Distance Quantum Communication with Atomic Ensembles

553

*C. W. Chou, S. V. Polyakov, D. Felinto, H. de Riedmatten,
S. J. van Enk and H. J. Kimble*

1	Introduction	553
2	DLCZ Protocol for Quantum Repeaters	555
3	Nonclassical Photon Pairs from an Atomic Ensemble	560
4	Atomic Ensemble as Conditional Source of Single Photons	564
5	Temporal Structure of the Nonclassical Correlations	566
6	Decoherence in the Atomic Ensemble	569
7	Prospect for Entanglement between Distant Ensembles	573
7.1	Single photon non-locality	575
7.2	Quantum tomography	576
8	Concluding Remarks	577
	References	578

Chapter 27

Decoherence and Decoherence Suppression in Ensemble-Based Quantum Memories for Photons

581

M. Fleischhauer and C. Mewes

1	Introduction	581
2	Two-Mode Quantum Memory	584
3	Equivalence Classes of Storage States and Sensitivity to Decoherence	589
3.1	Individual reservoir interactions	589
3.2	Collective reservoir interactions	591
4	Decoherence Suppression and Decoherence-Free Subspaces	592

5 Summary	597
References	598
Index	601