

# CHAPTER 1

## MANAGING DERIVATIVES IN THE PRESENCE OF A SMILE EFFECT AND INCOMPLETE INFORMATION

*Mondher Bellalah\**

*This chapter develops a simple option pricing model when markets can make sudden jumps in the presence of incomplete information. Incomplete information can be defined in the context of Merton's (1987) model of capital market equilibrium with incomplete information. In this context, analytic formulas can be derived for options using the Black–Scholes (1973) approach as in Bellalah (1999). The option value depends upon the probability and magnitude of jumps and a continuous volatility. The model is useful in explaining the smile effect and in extracting information costs. The model can be applied to hedging strategies for different strike prices and can be used for the valuation of different types of options.<sup>a</sup> It can also be used in the identification of mispriced options. Some simulations are run with and without shadow costs of incomplete information. We run some simulations to extract information costs using market data. Our model can be used to estimate information costs in different markets.*

### 1 Introduction

This chapter develops a simple option pricing model when markets can make sudden jumps in the presence of incomplete information. We build on Derman *et al.* (1991) modeling of jumps on the underlying asset and combine it with the Bellalah (1999) approach to include information costs.

---

\*THEMA, University of Cergy and ISC Paris.

<sup>a</sup>Many thanks to Riva F, for his help in running simulations.

These costs are defined with respect to Merton's (1987) simple model of capital market equilibrium with incomplete information: investors spend time and money to gather information about the financial instruments and financial markets.

The structure of the chapter is as follows. Section 2 explains the role of information costs in asset pricing and option pricing with respect to Merton's model of capital market equilibrium with incomplete information. In Sec. 3, we present the model we use for the valuation of option prices on the S&P 500 index when prices can jump and information costs are taken into account. The results of our simulations are presented in Sec. 4. Section 5 summarizes and concludes the chapter.

## 2 Option Pricing in the Presence of Information Costs

Differences in information can explain some puzzling phenomena in finance such as the "home equity bias" or the "weekend effect." Information costs can also offer an explanation for limited participation in financial markets. In general, a fixed cost to participate in the stock market is viewed as summarizing both transaction (as brokerage fees) and information costs (such as the cost of understanding financial institutions, the cost of gathering information about assets, etc.).

Merton (1987) adopts most of the assumptions of the original Capital Asset Pricing Model (CAPM) and relaxes the assumption of equal information across investors. Besides, he assumes that investors hold only securities of which they are aware. This assumption is motivated by the observation that portfolios held by actual investors include only a small fraction of all available traded securities.

The story of information costs applies in varying degrees to the adoption in practice of new structural models of evaluation, i.e. option pricing models. It applies also to the diffusion of innovations for several products and technologies. The recognition of the different speeds of information diffusion is particularly important in explaining the behavior of different firms.

In Merton's model, the expected returns increase with systematic risk, firm-specific risk, and relative market value. The expected returns decrease with relative size of the firm's investor base, referred to in Merton's model as the "degree of investor recognition".

The analysis of investment opportunities can be done in a standard option framework "à la Black-Scholes" (1973). These authors derive their model under the assumption that investors create riskless hedges between options

and their underlying securities. Besides, their formula relies implicitly on the CAPM.

Merton's model may be stated as follows:

$$R_S - r = \beta_S[R_m - r] + \lambda_S - \beta_S\lambda_m,$$

where

$R_S$ : the equilibrium expected return on an asset  $S$ ;

$R_m$ : the equilibrium expected return on the market portfolio;

$r$ : the riskless rate of interest;

$\beta_S$ :  $\text{cov}(R_S/R_m)/\text{var}(R_m)$ ;

$\lambda_S$ : the equilibrium aggregate "shadow cost" for the asset  $S$ , which is of the same dimension as the expected rate of return on the asset  $S$ ; and

$\lambda_m$ : the weighted average shadow cost of incomplete information over all assets.

Bellalah and Jacquilat (1995) and Bellalah (1999) provide a valuation formula for commodity options in the context of incomplete information. Their analysis is based on Merton's (1987) model and can be used to extend the analysis by Derman *et al.* (1991). This is the goal of the following section.

### 3 Valuing Options When Markets Can Jump in the Presence of Shadow Costs of Incomplete Information

We first briefly present how to integrate market jumps in a simple way and then extend the analysis to take into account information costs.

#### 3.1 Valuing Options When Market Can Jump

Consider the following simple model proposed by Derman *et al.* (1991). The underlying asset price at time 0 today is  $S$ . In the next instant, the underlying asset price can jump up by  $u\%$  to  $S_u$  with probability  $w$  or down by  $d\%$  to  $S_d$  with probability  $(1 - w)$ .

The probability  $w$  is expected to be close to 0 or 1. This means that either a jump up or a jump down predominates. After the first jump, the underlying asset will diffuse with constant volatility  $\sigma$  as in the Black–Scholes (1973) model. No other jumps will occur.

The value of any security in this model can be computed as the average of its payoffs over the scenarios where the underlying asset jumps up or down.

Hence, the option value is given by

$$\text{Option} = w\text{BS}(S_u, K, \sigma, r, \delta, T) + (1 - w)\text{BS}(S_d, K, \sigma, r, \delta, T), \quad (1)$$

where  $\text{BS}(S, K, \sigma, r, \delta, T)$  is the formula by Black–Scholes (1973) and  $\delta$  refers to the continuous dividend yield. This is the formula that appears in the work by Derman *et al.* (1991).

The values used for the underlying asset are:

$$S_u = S(1 + u); \quad S_d = S(1 - d).$$

The current value of the underlying asset corresponds also to an average value after a jump up and a jump down. Hence, the jump up and the jump down are related by

$$d(1 - w) = wu.$$

### 3.2 Extension with Information Costs

The extension of the jump model in the presence of shadow costs can be easily done. The value of any security in this model can be computed as the average of its payoffs over the scenarios where the underlying asset jumps up or down. This process corresponds to a continuous diffusion which is accompanied occasionally by a jump. The use of the Black–Scholes (1973) model assumes that all future variation in the underlying asset value is attributed to the continuous diffusion and none to the discontinuous jump.

The jump-diffusion process is defined by a diffusion volatility and a probability and magnitude for the discontinuous jump. The diffusion volatility characterizes the continuous diffusion. A small probability of a jump of the underlying asset price in the direction of the strike price can affect the value of an out-of-the-money option. In the presence of such a process, two options at least are necessary to extract information about the implied volatility and the implied jump. The model parameters are such that the model error, i.e. the sum of the squared difference between the model prices and the market prices for the two options are as close as possible to zero.

The same approach can be extended to allow the estimation of implied information costs from market data.

In our analysis, the option value is given by

$$\begin{aligned} \text{option} = & w\text{BS}(S_u, K, \sigma, r, \delta, \lambda_s, \lambda_c, T) \\ & + (1 - w)\text{BS}(S_d, K, \sigma, r, \delta, \lambda_s, \lambda_c, T), \end{aligned} \quad (2)$$

where  $\text{BS}(S_u, K, \sigma, r, \delta, \lambda_s, \lambda_c, T)$  is the formula given by Bellalah (1999).

In this context, the call value is given by

$$C = S \exp((\lambda_s - \lambda_c)T)N(d_1) - E \exp(-(r + \lambda_c)T)N(d_2), \quad (3)$$

$$d_1 = [\ln(S/E) + (r + \lambda_s + 1/2\sigma^2)T]/\sigma\sqrt{T}, \quad d_2 = d_1 - \sigma\sqrt{T},$$

where

- $S$ : the underlying asset price;
- $E$ : the strike price;
- $\lambda_s$ : the information cost on the asset  $S$ ;
- $\lambda_c$ : the information cost on the asset  $C$ ;
- $T$ : the time to maturity;
- $r$ : the riskless interest rate; and
- $\sigma$ : the volatility of the underlying asset.

For a derivation of this formula, the reader can refer to Bellalah (1990, 1999).

## 4 The Smile Effect and the S&P 500 Index Options in the Presence of Jumps and Incomplete Information

### 4.1 *The Smile*

Consider the implied volatilities on a given day for the European-style July S&P index options expiring with a given maturity. Table 1 shows the implied volatilities and the deltas of S&P calls and puts using the Black–Scholes (1973) model.

The option maturity date is in March 2001, the index level is 1264.74, the riskless interest rate is 5.81%, and the dividend yield is 1.17%. Note that the sign  $-$  refers to the put's delta and the sign  $+$  refers to the call's delta.

It is important to note that options with strike prices below the index price or out-of-the-money puts with low deltas are traded at higher implied volatilities than options with strike prices above the asset price which correspond to out-of-the-money calls with low deltas. The presence of different implied volatilities for different strike prices refers to the well-known smile. This may be viewed as an “anomaly” in the Black–Scholes model since when using their formula, one must adjust the volatility as the strike price changes. Besides, the fact that implied volatilities seem to be higher for puts than calls may be a “strange” result.

**Table 1:** Implied volatilities and the deltas of S&P calls and puts using the Black–Scholes (1973) model.

Strike	Type	$\sigma$ implied (%)	$\Delta$ (%)
950	Put	35.49	-0.37
975	Put	34.84	-0.72
1025	Put	34.72	-2.26
1050	Put	32.51	-3.68
1100	Put	31.84	-8.44
1125	Put	30.38	-11.97
1150	Put	29.80	-16.31
1175	Put	29.07	-21.45
1200	Put	28.21	-27.29
1250	Put	25.71	-40.45
1275	Call	24.28	52.42
1300	Call	24.57	45.56
1325	Call	23.65	38.96
1350	Call	22.47	32.77
1375	Call	22.02	27.13
1400	Call	21.20	22.10

#### 4.2 Introducing Market Jumps

In fact, if market participants believe that the underlying asset is driven by a continuous random walk, then the volatility must be independent of the strike price. This strange result can be explained by the fact that market participants expect an occasionally sharp downward jump in the underlying asset price. If it were the case, then out-of-the-money puts could exhibit a higher probability of paying off than out-of-the-money calls. In this case, the smile can be explained by a jump-diffusion process.

Using the market prices of at least two options on the same underlying asset and maturity with different strike prices, the Derman *et al.* (1991) model can be used to extract the market implied volatility and information regarding the implied jumps. Knowledge about the jump probability is necessary for the estimation. As mentioned above this probability is expected to be close to 0 here as data are consistent with expectations about a downward jump. In the Derman *et al.* (1991), model, the probability is explicitly chosen by the user. We take the same approach here but we also consider the possibility for  $w$  to be endogenously determined.

**Table 2:** Parameter estimates using the Derman *et al.* (1991) methodology.

$w$ (%)	$d$ (%)	$\sigma$ diffusion (%)
3	58.71	19.73
4	46.87	19.51
5	39.73	19.28
6	34.94	19.04
7	31.49	18.80
8	28.90	18.55
9	26.85	18.29
10	25.19	18.03
11	23.82	17.76
12	22.67	17.47
13	21.67	17.19
14	20.79	16.89
15	20.03	16.58

**Table 3:** Parameter estimates using the Derman *et al.* (1991) methodology with endogenous  $w$  parameter.

$w$	$d$	$\sigma$ diffusion
15.51%	19.67%	16.43%

The calibration has been made using the 1200 and the 1250 put options as they correspond to the most liquid options given the maturity we considered. The results are given in Tables 2 and 3. In Table 2, the results are based on direct application of the Derman *et al.* (1991) methodology, i.e. the  $w$  parameter value has been explicitly chosen. We give the results for a set of reasonable values, starting from  $w = 3\%$  as the algorithm was unable to achieve convergence for values less than this figure. An interesting result here is that the implied diffusion parameter is relatively insensitive to the  $w$  value which makes the model reliable even in the presence of error in  $w$  estimation by a trader. In Table 3, the  $w$  value is endogenously determined, i.e. we let the algorithm calculate the parameter values ( $w$ ,  $d$ , and  $\sigma$  diffusion) which best fit the market prices used for calibration. In the remainder of the chapter, we decided to restrict ourselves to this approach.

**Table 4:** Comparison between Black–Scholes and model prices.

Strike	Type	Market price	Black–Scholes price	Model price	Market $\sigma$ (%)	Model $\sigma$ (%)
950	Put	1.875	0.10	0.813	35.49	31.20
975	Put	2.625	0.21	1.587	34.84	31.90
1025	Put	5.750	0.87	4.512	34.72	32.84
1050	Put	6.375	1.59	6.746	32.51	32.98
1100	Put	12.000	4.60	12.630	31.84	32.38
1125	Put	14.500	7.28	16.194	30.38	31.64
1150	Put	18.875	11.05	20.186	29.80	30.66
1175	Put	24.000	16.13	24.719	29.07	29.49
1200	Put	30.000	22.73	30.012	28.21	28.22
1250	Put	44.125	41.07	44.140	25.71	25.71
1275	Call	54.000	54.00	54.704	24.28	24.60
1300	Call	43.625	42.98	41.540	24.57	23.62
1325	Call	32.375	33.70	30.549	23.65	22.78
1350	Call	22.500	26.03	21.727	22.47	22.06
1375	Call	15.875	19.80	14.930	22.02	21.46
1400	Call	10.250	14.84	9.907	21.20	20.96

Table 4 gives a comparison between the market price, the Black–Scholes price, and the model price,<sup>b</sup> and between the model implied diffusion volatility and the Black–Scholes implicit volatility. Of course, the model volatility is less than the Black–Scholes volatility as the model makes a correction for the implicit jump conveyed by the prices. Note that the model gives a partial correction for the bias one can observe when applying the Black–Scholes formula for out-of-the-money calls and puts.

### 4.3 Introducing Information Costs

We introduce information costs in the Derman *et al.* (1991) methodology. We considered information costs both on the option market ( $\lambda_c$ ), and the underlying asset ( $\lambda_s$ ), and ran simulations for different cost levels (from 1% to 5%). However, due to space considerations, we restrict our presentation in Fig. 1 to the most significant results. We decided to compare the model price and the market price in terms of implied volatility in order to exhibit the model ability to fit the existing smile.

<sup>b</sup>The input value of sigma for the Black–Scholes formula has been estimated using the 1250 at-the-money put.

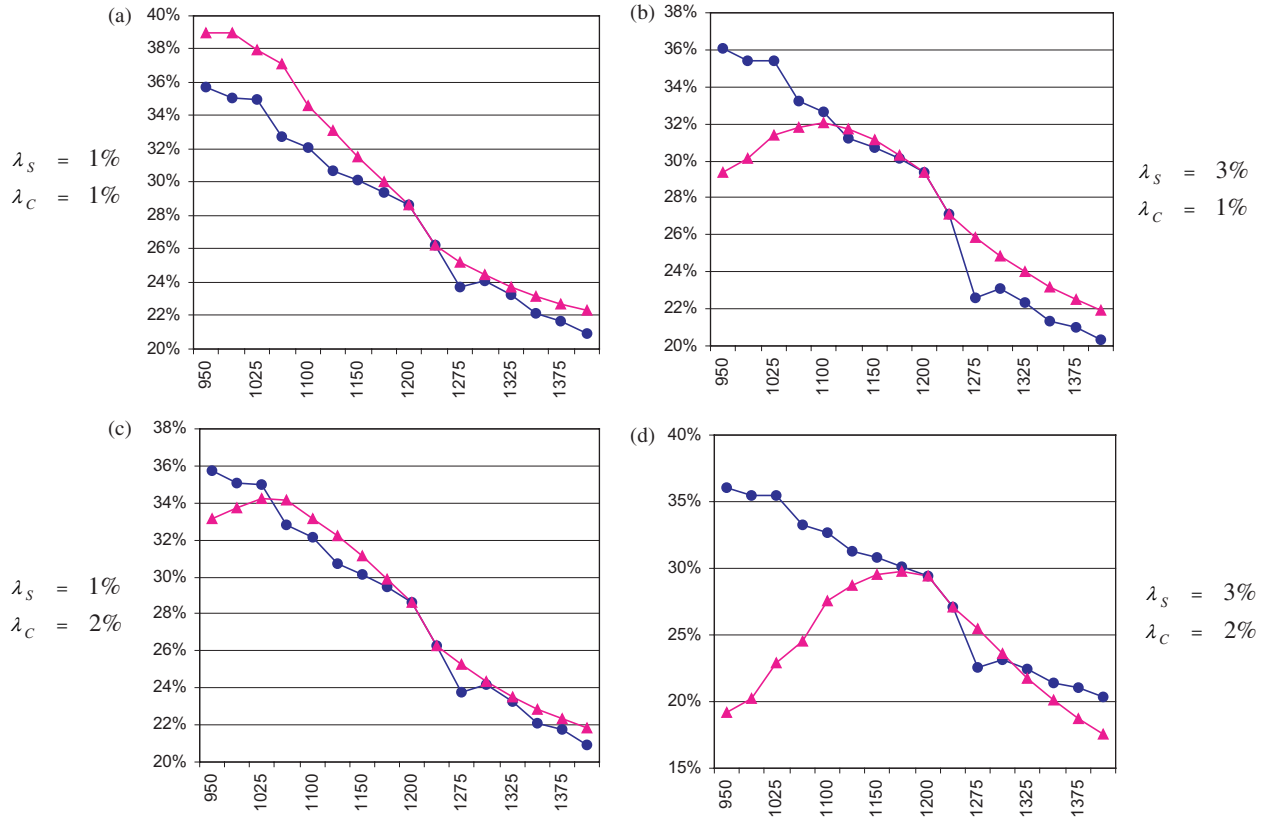


Figure 1: Comparison of implied volatility between market prices and model prices for different information cost levels.

One can notice at first glance that the introduction of information costs makes possible the production of any smile pattern (see, for example, the differences between panels a and d) which makes information costs a promising tool for explaining the volatility smile. Another striking aspect is that the information cost levels which give the best fitting ( $\lambda_s = 1\%$  and  $\lambda_c = 2\%$ ) are very close to Merton's estimates although we use a radically different approach. Thus, we can view our model as a possible (and reliable) way to extract information costs using option prices.

## 5 Summary and Conclusion

This chapter develops a simple model for the valuation of options in the presence of jumps and information costs. The model is an extension of the models of Derman *et al.* (1991) and Bellalah (1999). Our model has the potential to explain the smile effect. It is calibrated to market data and allows an implicit estimation of the magnitude of information costs. While our methodology and our model are applied only to index options, they can be used in different option markets.

## References

- Bellalah, M (1990). Quatres Essais Sur L'évaluation des Options: Dividendes, Volatilités des Taux d'intérêt et Information Incomplète, Doctorat de l'université de Paris-Dauphine.
- Bellalah, M and Jacquillat, B (1995). Option valuation with information costs: Theory and Tests. *Financial Review*, August, 617–635.
- Bellalah, M (1999). The valuation of futures and commodity options with information costs. *Journal of Futures Markets*, September.
- Black, F and Scholes, M (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, 81, 637–659.
- Derman, E, Bergier A and Kani, I (1991). Valuing index options when markets can jump. Working paper, Quantitative Strategies Research Notes, Goldman Sachs, July.
- Merton, RC (1987). A simple model of capital market equilibrium with incomplete information, *Journal of Finance*, 42, 483–510.